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Properties of the solution of some multipoint problem for integro-differential equation of Barbashin type

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Abstract. The nonlocal boundary value problem for the nonlinear integro-differential Barbashin equation with a partial derivative of the second order is studied. The solution of the problem is understood in the classical sense. The problem reduces to the equivalent Volterra integral equation with partial integrals. The conditions for the existence and uniqueness of the solution of this equation are obtained.

Keywords. Nonlinear integro-differential Barbashin equation, nonlocal boundary value problem, existence and uniqueness, Volterra integral equation with partial integrals, classical solution.

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1 Introduction

Boundary value problems for Barbashin or similar equations arise in the mathematical modeling of various phenomena of transport theory, e.g., in the propagation of radiation through the atmosphere of planets and stars, the transfer of neutrons through thin plates and membranes in nuclear reactors, and in several other transport problems and are studied by many scientists (see, for example, [1] and the references therein). In the paper [2], the Cauchy problem

$$\begin{cases} \dfrac{\partial^2 x(t,s)}{\partial t^2} = f\left(t,s,x(t,s)\right) + \displaystyle\int_c^d n\left(t,s,\delta,x(t,\delta)\right) d\delta + g(t,s), \\ a < t < b,\ c \leq s \leq d, \\ x(a,s) = \phi(s),\ x_t(a,s) = \psi(s),\ c \leq s \leq d \end{cases}$$

for the nonlinear integro-differential Barbashin equation with a partial derivative of the second order was studied. The problem reduces to the equivalent Volterra integral equation with partial integrals. The conditions for the existence and uniqueness of the solution of this equation were obtained. A theorem on the numerical solution of the nonlinear integral Volterra equation with partial integrals was given.

In the present paper, the nonlocal boundary value problem

$$\begin{cases} \dfrac{\partial^2 u(x,t)}{\partial t^2} + a\dfrac{\partial u(x,t)}{\partial t} = f\left(x,t,u(x,t)\right) + \displaystyle\int_0^l K\left(x,t,\tau,u(\tau,t)\right)d\tau, \\ (x,t) \in [0,l] \times (0,T) = D, \\ u(x,0) + \displaystyle\sum_{i=1}^{P} \alpha_i u\left(x,t_i\right) = \phi(x), \\ \dfrac{\partial u(x,0)}{\partial t} + \displaystyle\sum_{i=1}^{P} \beta_i \dfrac{\partial u\left(x,t_i\right)}{\partial t} = \psi(x),\ x \in [0,l], \\ 0 < t_1 \le t_2 \le \cdots \le t_P \le T \end{cases} \tag{1}$$

for the nonlinear integro-differential Barbashin equation with a partial derivative of the second order is investigated. The solution of the problem is understood in the classical sense. The problem reduces to the equivalent Volterra integral equation with partial integrals. The sufficient conditions are obtained for the existence and uniqueness of the solution of problem (1). The theorem on the continuous dependence of solution on parameters is established. Finally, nonlocal boundary value problems for the linear parabolic, elliptic equations and equations of mixed types have been studied extensively (see, for instance, [3–12] and the references therein).

2 Existence and uniqueness of solution of problem (1)

Let

$$1 + \sum_{i=1}^{P} \alpha_i \left(1 + at_i\right) = A \neq 0,\ \ 1 + \sum_{i=1}^{P} \beta_i \left(1 - at_i\right) = B \neq 0$$

and

$$1 + \frac{a^2}{AB} \sum_{i=1}^{P} \alpha_i t_i \sum_{j=1}^{P} \beta_j t_j = C \neq 0.$$

Theorem 2.1. *Suppose that the functions $\phi, \psi \in C[0,l]$, $f \in C\left(D, \mathfrak{R}^1\right)$, $K \in C\left(D, [0,l], \mathfrak{R}^1\right)$ and for all $(x,t) \in D, \tau \in [0,l], u, \vartheta \in \mathfrak{R}^1$ the following conditions hold*

$$\begin{gathered} |f(x,t,u) - f(x,t,\vartheta)| \le L_1 |u - \vartheta|, \\ |K(x,t,\tau,u) - K(x,t,\tau,\vartheta)| \le L_2 |u - \vartheta|, \end{gathered} \tag{2}$$

where $L_1, L_2 = const \geq 0$. *Moreover, let*

$$d = \max_{(x,t)\in D} \left| g(x,t) + \sum_{i=1}^{P} \int_0^{t_i} q\left(s,t,t_i,\alpha_i,\beta_i\right) f(x,s,0)ds \right.$$

$$+ \sum_{i=1}^{P} \int_0^{t_i} \int_0^{l} q\left(s,t,t_i,a_i,\beta_i\right) K(x,s,\tau,0)d\tau ds + \int_0^t (t-s)f(x,s,0)ds$$

$$\left. + \int_0^t \int_0^l (t-s)K(x,s,\tau,0)d\tau ds \right| < \infty, \tag{3}$$

where

$$g(x,t) = \frac{\phi(x)}{A}\left((1+at)\left(1-\frac{a^2}{ABC}\sum_{i=1}^{P}\alpha_i t_i \sum_{j=1}^{P}\beta_j t_j\right) + \frac{a^2 t}{BC}\sum_{i=1}^{P}\beta_i t_i\right)$$

$$+\frac{\psi(x)}{C}\left(t-\frac{1}{A}\sum_{i=1}^{P}\alpha_i t_i\right), \tag{4}$$

$$h\left(t,\alpha_i,\beta_i\right)$$

$$= a\left(\frac{(1+at)\alpha_i}{A} + \frac{a}{BC}\left(\frac{a\alpha_i}{A}\sum_{j=1}^{P}\beta_j t_j - \beta_i\right)\left(t-\frac{1+at}{A}\sum_{j=1}^{P}\alpha_j t_j\right)\right), \tag{5}$$

$$q\left(s,t,t_i,\alpha_i,\beta_i\right) = \alpha_i\left(t_i - s\right)$$

$$\times\left(\frac{a^2}{ABC}\left((1+at)\sum_{j=1}^{P}\alpha_j t_j - t\sum_{j=1}^{P}\beta_j t_j\right) - \frac{(1+at)}{A}\right)$$

$$+\beta_i\left(1-a\left(t_i-s\right)\right)\left(\frac{1}{BC}\left(\sum_{j=1}^{P}\alpha_j t_j - t\right)\right). \tag{6}$$

If there is $\lambda = const > 0$ *satisfying the inequality*

$$\beta = \lambda^2\left\|h\right\| + \left(\lambda L_1 + L_2\right)\left\|q\right\|\sum_{i=1}^{P}\left(e^{\lambda t_i}-1\right) + \lambda|a|\left(1-e^{-\lambda T}\right)$$

$$+L_1\left(1-e^{-\lambda T}-\lambda T\right) + L_2\left(e^{\lambda T}-1\right)\left(1-e^{-\lambda T}-\lambda T\right) < \lambda^2, \tag{7}$$

then there exists a unique solution $u(x,t) \in C\left(D\right)$ *of problem* (1) *with* $u_{tt} \in C\left(D\right)$.

Proof. This theorem makes it possible to reduce the existence and uniqueness of problem (1) to an equivalent integral equation. To do this, we will integrate equation (1) with respect to t twice and apply nonlocal conditions. Actually, taking the integral with respect to s from 0 to t, we get

$$\frac{\partial u\left(x,t\right)}{\partial t}=\frac{\partial u\left(x,0\right)}{\partial t}-au\left(x,t\right)+au\left(x,0\right)$$

$$+\int_0^t f\left(x,s,u\left(x,s\right)\right)ds+\int_0^t\int_0^l K\left(x,s,\tau,u\left(\tau,s\right)\right)d\tau ds.$$

Integrating the last equation and transforming the double integral with single variable integral, we get

$$u(x,t)=u(x,0)+\frac{\partial u(x,0)}{\partial t}t-a\int_0^t u(x,s)ds$$

$$+atu(x,0)+\int_0^t (t-s)f\left(x,s,u(x,s)\right)ds$$

$$+\int_0^t\int_0^l (t-s)K\left(x,s,\tau,u(\tau,s)\right)d\tau ds.$$

Applying these equations, we get

$$\frac{\partial u(x,t)}{\partial t}=\frac{\partial u(x,0)}{\partial t}-au(x,0)-at\frac{\partial u(x,0)}{\partial t}+a^2\int_0^t u(x,s)ds$$

$$-a^2tu(x,0)-a\int_0^t (t-s)f\left(x,s,u(x,s)\right)ds$$

$$-a\int_0^t\int_0^l (t-s)K\left(x,s,\tau,u(\tau,s)\right)d\tau ds$$

$$+au(x,0)+\int_0^t f\left(x,s,u(x,s)\right)ds+\int_0^t\int_0^l K\left(x,s,\tau,u(\tau,s)\right)d\tau ds$$

or

$$\frac{\partial u\left(x,t\right)}{\partial t}=\left(1-at\right)\frac{\partial u\left(x,0\right)}{\partial t}-a^2tu\left(x,0\right)+a^2\int_0^t u\left(x,s\right)ds$$

$$+\int_0^t \left(1-a\left(t-s\right)\right)f\left(x,s,u\left(x,s\right)\right)ds$$

$$+\int_0^t\int_0^l \left(1-a\left(t-s\right)\right)K\left(x,s,\tau,u\left(\tau,s\right)\right)d\tau ds.$$

Using given nonlocal conditions, we obtain the following integral equation

$$u(x,t) = g(x,t) + \sum_{i=1}^{P} \int_0^{t_i} h\left(t, \alpha_i, \beta_i\right) u(x,s) ds$$

$$+ \sum_{i=1}^{P} \int_0^{t_i} q\left(s, t, t_i, \alpha_i, \beta_i\right) f\left(x, s, u(x,s)\right) ds$$

$$+ \sum_{i=1}^{P} \int_0^{t_i} \int_0^{l} q\left(x, t, t_i, \alpha_i, \beta_i\right) K\left(x, s, \tau, u(\tau, s)\right) d\tau ds$$

$$-a \int_0^t u(x,s) ds + \int_0^t (t-s) f\left(x, s, u(x,s)\right) ds$$

$$+ \int_0^t \int_0^l (t-s) K\left(x, s, \tau, u(\tau,s)\right) d\tau ds,$$

where functions $g(x,t)$, $h\left(t, \alpha_i, \beta_i\right)$ and $q\left(s, t, t_i, \alpha_i, \beta_i\right)$ are defined by formulas (4), (5) and (6), respectively. So, we have that

$$u(x,t) = \Phi u(x,t), \tag{8}$$

where integral operator Φ is defined by formula

$$\Phi u(x,t) = g(x,t) + \sum_{i=1}^{P} \int_0^{t_i} h\left(x, \alpha_i, \beta_i\right) u(x,s) ds$$

$$+ \sum_{i=1}^{P} \int_0^{t_i} q\left(x, t, t_i, \alpha_i, \beta_i\right) f\left(x, s, u(x,s)\right) ds$$

$$+ \sum_{i=1}^{P} \int_0^{t_i} \int_0^{l} q\left(x, t, t_i, \alpha_i, \beta_i\right) K\left(x, s, \tau, u(\tau, s)\right) d\tau ds$$

$$-a \int_0^t u(x,s) ds + \int_0^t (t-s) f\left(x, s, u(x,s)\right) ds$$

$$+ \int_0^t \int_0^l (t-s) K\left(x, s, \tau, u(\tau,s)\right) d\tau ds.$$

It is clear that $\Phi : C(D) \to C(D)$. Moreover, Φ is a contraction mapping on $C(D)$ with the norm defined by

$$\|u\|_* = \max_{(x,t)\in D} e^{-\lambda(x+t)} \left|u(x,t)\right|, \ \lambda = const > 0.$$

Applying the triangle inequality, we get

$$|\Phi u(x,t)-\Phi v(x,t)| \leq \sum_{i=1}^{P}\int_0^{t_i}|h\left(x,\alpha_i,\beta_i\right)|\,|u(x,s)-\vartheta(x,s)|\,ds$$

$$+\sum_{i=1}^{P}\int_0^{t_i}|q\left(x,t,\alpha_i,\beta_i\right)|\,|f(x,s,u\;(\;x,s)-f\;(x,s,\vartheta(x,s))|\,ds$$

$$+\sum_{i=1}^{P}\int_0^{t_i}\int_0^{l}|q\left(x,t,t_i,\alpha_i,\beta_i\right)|\,|K\left(x,s,\tau,u(\tau,s)\right)-K\left(x,s,\tau,\vartheta(\tau,s)\right)|\,d\tau ds$$

$$+|a|\int_0^{t}|u(x,s)-\vartheta(x,s)|\,ds$$

$$+\int_0^{t}(t-s)\,|f\left(x,s,u(x,s)\right)-f\left(x,s,\vartheta(x,s)\right)|\,ds$$

$$+\int_0^{t}\int_0^{l}(t-s)\,|K\left(x,s,\tau,u(\tau,s)\right)-K\left(x,s,\tau,\vartheta(\tau,s)\right)|\,d\tau ds$$

$$\leq\left(\|h\|+\|q\|\,L_1\right)\sum_{i=1}^{P}\int_0^{t_i}|u(x,s)-\vartheta(x,s)|\,ds$$

$$+\|q\|\,L_2\sum_{i=0}^{P}\int_0^{t_i}\int_0^{l}|u(\tau,s)-\vartheta(\tau,s)|\,d\tau ds+|a|\int_0^{t}|u(x,s)-\vartheta(x,s)|\,ds$$

$$+L_1\int_0^{t}(t-s)\,|u(x,s)-\vartheta(x,s)|\,ds+L_2\int_0^{t}\int_0^{l}|u(\tau,s)-\vartheta(\tau,s)|\,d\tau ds$$

for all $u(x,t),\vartheta(x,t)\in C(D)$. Therefore,

$$\|\Phi u-\Phi v\|_{*}=\max_{(x,t)\in D}e^{-\lambda(x+t)}\,|\Phi u(x,t)-\Phi(x,t)|$$

$$\leq\max_{(x,t)\in D}e^{-\lambda(x+t)}\left\{\left(\|h\|+L_1\,\|q\|\right)\sum_{i=1}^{P}\int_0^{t_i}|u(x,s)-\vartheta(x,s)|\,ds\right.$$

$$+L_2\,\|q\|\sum_{i=1}^{P}\int_0^{t_i}\int_0^{l}|u(\tau,s)-\vartheta(\tau,s)|\,d\tau ds+|a|\int_0^{t}|u(x,s)-\vartheta(x,s)|\,ds$$

$$\left.+L_1\int_0^{t}(t-s)\,|u(x,s)-\vartheta(x,s)|\,ds+L_2\int_0^{t}\int_0^{l}|u(\tau,s)-\vartheta(\tau,s)|\,d\tau ds\right\}$$

$$\leq \frac{1}{\lambda^2}\left\{\lambda^2\|h\| + (\lambda L_1 + L_2)\|q\|\sum_{i=1}^{P}\left(e^{\lambda t_i} - 1\right) + \lambda|a|\left(1 - e^{-\lambda T}\right)\right.$$

$$+L_1\left(1 - e^{-\lambda T} - \lambda T\right)$$

$$\left.+L_2\left(e^{\lambda T} - 1\right)\left(1 - e^{-\lambda T} - \lambda T\right)\right\}\|u - \vartheta\|_* = \gamma\|u - \vartheta\|_*.$$

Here, $\gamma = \frac{\beta}{\lambda^2} < 1$. It means that Φ is a contraction mapping on $C(D)$. The assertion of the theorem now follows from the principle of contraction mapping. Theorem is proved.

3 Continuous dependence of solution on parameters

In this section, the nonlocal boundary value problem

$$\begin{cases} \frac{\partial^2 u(x,t)}{\partial t^2} + a\frac{\partial u(x,t)}{\partial t} = f\left(x, t, u(x,t), \mu\right) + \int\limits_0^l K\left(x, t, \tau, u(\tau, t), \nu\right) d\tau, \\ (x,t) \in [0,l] \times (0,T) = D, \\ u(x,0) + \sum\limits_{i=1}^{P} \alpha_i u\left(x, t_i\right) = \phi(x), \\ \frac{\partial u(x,0)}{\partial t} + \sum\limits_{i=1}^{P} \beta_i \frac{\partial u(x,t_i)}{\partial t} = \psi(x),\ x \in [0,l], \\ 0 < t_1 \leq t_2 \leq \cdots \leq t_P \leq T \end{cases} \tag{9}$$

for the nonlinear integro-differential Barbashin equation with a partial derivative of the second order is investigated. Here $\mu, \nu \in \mathfrak{R}^1$ are parameters.

Theorem 3.1. *Suppose that the functions $\phi, \psi \in C[0,l]$, $f \in C\left(D, \mathfrak{R}^1 \times \mathfrak{R}^1\right)$, $K \in C\left(D, [0,l], \mathfrak{R}^1 \times \mathfrak{R}^1\right)$ and for all $(x,t) \in D$, $\tau \in [0,l]$, $u, \vartheta, \mu, \nu \in \mathfrak{R}^1$ the following conditions hold*

$$|f(x,t,u,\mu) - f(x,t,\vartheta,\nu)| \leq L_1|u - \vartheta| + L_3|\mu - \nu|,$$

$$|K(x,t,\tau,u,\mu) - K(x,t,\tau,\vartheta,\nu)| \leq L_2|u - \vartheta| + L_4|\mu - \nu|,$$

where $L_i = const \geq 0$ $\left(i = \overline{1,4}\right)$. Moreover, let λ be a positive constant satisfying the inequality (7). Then, there exists a unique solution $u(x,t) \in C(D)$ of problem (9) with $u_{tt} \in C(D)$ and it depends continuously on parameters.

Proof. The existence and uniqueness of the solution of problem (9) follows from Theorem 2.1 for each value of the parameters μ and ν. To complete the proof of this theorem, it suffices to show that the solution depends continuously on the parameters. We have that

$$u(x,t) = g(x,t) + \sum_{i=1}^{P} \int_0^{t_i} h\left(t, \alpha_i, \beta_i\right) u(x,s) ds$$

$$+ \sum_{i=1}^{P} \int_0^{t_i} q\left(s, t, \alpha_i, t_i\right) f\left(x, s, u(x,s), \mu\right) ds$$

$$+ \sum_{i=1}^{P} \int_0^{t_i} \int_0^{l} q\left(x, t, \alpha_i, \beta_i\right) K\left(x, s, \tau, u(\tau, s), \nu\right) d\tau ds$$

$$-a \int_0^t u(x,s) ds + \int_0^t (t-s) f\left(x, s, u(x,s), \mu\right) ds$$

$$+ \int_0^t \int_0^l (t-s) K\left(x, s, \tau, u(\tau, s), \nu\right) d\tau ds. \tag{10}$$

Let us denote by $u_k(x,t)$ $(k = 1,2)$ the solutions of equation (10) for the respective $\mu = \mu_k$, $\nu = \nu_k$ parameters. Then,

$$u_k(x,t) = g(x,t) + \sum_{i=1}^{P} \int_0^{t_i} h\left(t, \alpha_i, \beta_i\right) u_k(x,s) ds$$

$$+ \sum_{i=1}^{P} \int_0^{t_i} q\left(s, t, t_i, \alpha_i, t_i\right) f\left(x, s, u_k(x,s), \mu_k\right) ds$$

$$+ \sum_{i=1}^{P} \int_0^{t_i} \int_0^{l} q\left(x, t, t_i, \alpha_i, \beta_i\right) K\left(x, s, \tau, u_k(\tau, s), \nu_k\right) d\tau ds$$

$$-a \int_0^t u_k(x,s) ds + \int_0^t (t-s) f\left(x, s, u_k(x,s), \mu_k\right) ds$$

$$+ \int_0^t \int_0^l (t-s) K\left(x, s, \tau, u_k(\tau, s), \nu_k\right) d\tau ds, \ k = 1,2.$$

Applying this formula and the triangle inequality, we get

$$|u_1(x,t) - u_2(x,t)| \leq \sum_{i=1}^{P} \int_0^{t_i} \left|h\left(t, \alpha_i, \beta_i\right)\right| |u_1(x,s) - u_2(x,s)| \, ds$$

$$+L_1 \sum_{i=1}^{P} \int_0^{t_i} |q(s,t,t_i,\alpha_i,t_i)| \, |u_1(x,s) - u_2(x,s)| \, ds$$

$$+L_2 \sum_{i=1}^{P} \int_0^{t_i} \int_0^{l} |q(x,t,t_i,\alpha_i,\beta_i)| \, |u_1(\tau,s) - u_2(\tau,s)| \, d\tau ds$$

$$+|a| \int_0^{t} |u_1(x,s) - u_2(x,s)| \, ds$$

$$+L_1 \int_0^{t} (t-s) \, |u_1(x,s) - u_2(x,s)| \, ds$$

$$+L_2 \int_0^{t} \int_0^{l} (t-s) \, |u_1(\tau,s) - u_2(\tau,s)| \, d\tau ds$$

$$+L_3 \, |\mu_1 - \mu_2| \sum_{i=1}^{P} \int_0^{t_i} |q(s,t,t_i,\alpha_i,t_i)| \, ds$$

$$+L_4 \, |\nu_1 - \nu_2| \sum_{i=1}^{P} \int_0^{t_i} \int_0^{l} |q(x,t,t_i,\alpha_i,\beta_i)| \, d\tau ds$$

$$+L_3 \, |\mu_1 - \mu_2| \int_0^{t} (t-s) ds + L_4 \, |\nu_1 - \nu_2| \int_0^{t} \int_0^{l} (t-s) d\tau ds$$

for all $u_1(x,t), u_2(x,t) \in C(D)$. Therefore,

$$\|u_1 - u_2\|_* = \max_{(x,t)\in D} e^{-\lambda(x+t)} \, |u_1(x,t) - u_2(x,t)|$$

$$\leq \max_{(x,t)\in D} e^{-\lambda(x+t)} \Big\{ (\|h\| + L_1 \|q\|) \sum_{i=1}^{P} \int_0^{t_i} |u_1(x,s) - u_2(x,s)| \, ds$$

$$+L_2 \|q\| \sum_{i=1}^{P} \int_0^{t_i} \int_0^{l} |u_1(\tau,s) - u_2(\tau,s)| \, d\tau ds + |a| \int_0^{t} |u_1(x,s) - u_2(x,s)| \, ds$$

$$+L_1 \int_0^{t} (t-s) \, |u_1(x,s) - u_2(x,s)| \, ds + L_2 \int_0^{t} \int_0^{l} |u_1(\tau,s) - u_2(\tau,s)| \, d\tau ds \Big\}$$

$$+L_3 \left(\|q\| \sum_{i=1}^{P} t_i + \frac{t^2}{2} \right) |\mu_1 - \mu_2| + L_4 \left(\|q\| \sum_{i=1}^{P} t_i + \frac{t^2}{2} \right) l \, |\nu_1 - \nu_2|$$

$$\leq \gamma \, \|u_1 - u_2\|_* + \|q\| \left(\sum_{i=1}^{P} t_i + \frac{T^2}{2} \right) (L_3 \, |\mu_1 - \mu_2| + L_4 l \, |\nu_1 - \nu_2|) \, .$$

From that it follows

$$\|u_1 - u_2\|_* \leq \frac{1}{1-\gamma}\left(\|q\|\sum_{i=1}^{P} t_i + \frac{T^2}{2}\right)\Big(L_3\left|\mu_1 - \mu_2\right| + L_4 l\left|\nu_1 - \nu_2\right|\Big).$$

The last inequality shows the continuity of the unique solution $u(x, t)$ of problem (9) in *D* with respect to parameters μ and ν. Theorem is proved.

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