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On $(f,\rho)-$Statistical convergence and strong $(f,\rho)-$summability of order α

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Abstract. The main object of this article is to introduce the concepts of $(f,\rho)-$ statistical convergence of order α and strong $(f,\rho)-$ summability of order α of sequences of real numbers and give some inclusion relations between these spaces.

Keywords. Modulus function, statistical convergence, $\rho-$ statistical convergence, strong $\rho-$summability.

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1 Introduction

In 1951, Steinhaus [29] and Fast [15] introduced the concept of statistical convergence and later in 1959, Schoenberg [28] reintroduced independently. Altın et al. [3], Bhardwaj and Dhawan [6], Çakallı ([8,9]), Caserta et al. [7], Çınar et al. [10], Çolak [11], Connor [12], Di Maio and Kočinac [13], Et et al. ([14,18,19,32,33]), Fridy [16], Aral and Şengül Kandemir [4], Işık and Akbaş ([2,20,21]), Salat [27], Başar et al. ([5,23]) and many authors investigated some arguments related to this notion.

A modulus f is a function from $[0,\infty)$ to $[0,\infty)$ such that

i) $f(x)=0$ if and only if $x=0$,

ii) $f(x+y)\leq f(x)+f(y)$ for $x,y\geq 0$,

iii) f is increasing,

iv) f is continuous from the right at 0.

It follows that f must be continuous in everywhere on $[0,\infty)$. A modulus may be unbounded or bounded.

Aizpuru et al. [1] defined $f-$density of a subset $E\subset\mathbb{N}$ for any unbounded modulus f by

$$d^f(E)=\lim_{n\to\infty}\frac{f(|\{k\leq n:k\in E\}|)}{f(n)},\text{ if the limit exists}$$

and defined $f-$statistical convergence for any unbounded modulus f by

$$d^f\left(\{k\in\mathbb{N}:|x_k-\ell|\geq\varepsilon\}\right)=0$$

and we write it as $S^f-\lim x_k=\ell$ or $x_k\rightarrow\ell\left(S^f\right)$. Every $f-$statistically convergent sequence is statistically convergent, but a statistically convergent sequence does not need to be $f-$statistically convergent for any unbounded modulus f.

A sequence (x_k) of points in $\mathbb{R}$ (the set of real numbers) is called $\rho-$statistically convergent to ℓ, if

$$\lim_{n\rightarrow\infty}\frac{1}{\rho_n}|\{k\leq n:|x_k-\ell|\geq\varepsilon\}|=0$$

for each $\varepsilon>0$, where $\rho=(\rho_n)$ is a non-decreasing sequence of positive real numbers tending to ∞ such that $\limsup_n\frac{\rho_n}{n}<\infty$ and $\Delta\rho_n=O(1)$, where $\Delta\rho_n=\rho_{n+1}-\rho_n$ for each positive integer n. In this case we write $S_\rho-\lim x_k=\ell$ or $x_k\rightarrow\ell\,(S_\rho)$. We denote the set of all $\rho-$statistically convergent sequences by S_ρ. If $\rho_n=n$ for each positive integer n, then $\rho-$statistical convergence is coincided statistical convergence [8].

The notion of a modulus was given by Nakano [24]. Maddox [22] used a modulus function to construct some sequence spaces. Afterwards different sequence spaces defined by modulus have been studied by Gaur and Mursaleen [17], Nuray and Savas [25], Pehlivan and Fisher [26], Şengül [31] and many others.

2 Main results

In this section we will introduce the concepts of $(f,\rho)-$statistically convergent sequences of order α and strongly $(f,\rho)-$summable sequences of order α of real numbers, where f is an unbounded modulus and give some inclusion relations between these concepts.

Definition 2.1. Let f be an unbounded modulus and α be a real number such that $0<\alpha\leq1$. We say that the sequence $x=(x_k)$ is $(f,\rho)-$statistically convergent of order α, if there is a real number ℓ such that

$$\lim_{n\rightarrow\infty}\frac{1}{f\left(\rho_n\right)^\alpha}f\left(|\{k\leq n:|x_k-\ell|\geq\varepsilon\}|\right)=0,$$

for each $\varepsilon>0$, where and afterwards $\rho=(\rho_n)$ is a non-decreasing sequence of positive real numbers tending to ∞ such that $\limsup_n\frac{\rho_n}{n}<\infty$, $\Delta\rho_n=O(1)$, and $\Delta\rho_n=\rho_{n+1}-\rho_n$ for each positive integer n. $f\left(\rho_n\right)^\alpha$ denotes the αth power of

$f(\rho_n)$, that is $(f(\rho_n)^\alpha) = (f(\rho_1)^\alpha, f(\rho_2)^\alpha, ..., f(\rho_n)^\alpha, ...)$. In this case, we write $S_\rho^{f,\alpha} - \lim x_k = \ell$ or $x_k \to \ell \left(S_\rho^{f,\alpha}\right)$. This space will be denoted by $S_\rho^{f,\alpha}$. In case of $\rho_n = 1$, for all $n \in \mathbb{N}$, we write $S^{f,\alpha}$ instead of $S_\rho^{f,\alpha}$.

Definition 2.2. Let f be a modulus function and α be a real number such that $0 < \alpha \leq 1$. We say that the sequence $x = (x_k)$ is strongly $w^\alpha[\rho, f, p]$ −summable to ℓ (a real number), if there is a real number ℓ such that

$$\lim_{n\to\infty} \frac{1}{\rho_n^\alpha} \sum_{k=1}^{n} [f(|x_k - \ell|)]^{p_k} = 0,$$

and is denoted by $w^\alpha[\rho, f, p] - \lim x_k = \ell$. The set of all strongly $w^\alpha[\rho, f, p]$ −summable sequences will be denoted by $w^\alpha[\rho, f, p]$. In case of $p_k = 1$, for all $k \in \mathbb{N}$, we write $w^\alpha[\rho, f]$ instead of $w^\alpha[\rho, f, p]$.

Definition 2.3. Let f be an unbounded modulus and α be a real number such that $0 < \alpha \leq 1$. We say that the sequence $x = (x_k)$ is strongly $w_\rho^{f,\alpha}(p)$ −summable to ℓ (a real number), if there is a real number ℓ such that

$$\lim_{n\to\infty} \frac{1}{f(\rho_n)^\alpha} \sum_{k=1}^{n} [f(|x_k - \ell|)]^{p_k} = 0,$$

and is written as $w_\rho^{f,\alpha}(p) - \lim x_k = \ell$. The set of all strongly $w_\rho^{f,\alpha}(p)$ −summable sequences will be denoted by $w_\rho^{f,\alpha}(p)$. In case of $p_k = 1$, for all $k \in \mathbb{N}$, we write $w_\rho^{f,\alpha}$ instead of $w_\rho^{f,\alpha}(p)$ and in case of $p_k = p$ for all $k \in \mathbb{N}$ we write $w_\rho^{f,\alpha}[p]$ instead of $w_\rho^{f,\alpha}(p)$.

Definition 2.4. Let f be an unbounded modulus and α be a real number such that $0 < \alpha \leq 1$. We say that the sequence $x = (x_k)$ is strongly $w_{\rho,f}^\alpha(p)$ −summable to ℓ (a real number), if there is a real number ℓ such that

$$\lim_{n\to\infty} \frac{1}{f(\rho_n)^\alpha} \sum_{k=1}^{n} |x_k - \ell|^{p_k} = 0,$$

and is written as $w_{\rho,f}^\alpha(p) - \lim x_k = \ell$. The set of all strongly $w_{\rho,f}^\alpha(p)$ −summable sequences will be denoted by $w_{\rho,f}^\alpha(p)$. In case of $p_k = 1$, for all $k \in \mathbb{N}$, we write $w_{\rho,f}^\alpha$ instead of $w_{\rho,f}^\alpha(p)$ and in case of $p_k = p$ for all $k \in \mathbb{N}$ we write $w_{\rho,f}^\alpha[p]$ instead of $w_{\rho,f}^\alpha(p)$.

The proof of each of the following results is fairly straightforward, so we choose to state these results without proof, where we shall assume that the sequence $p=(p_k)$ is bounded and $0<h=\inf_k p_k \le p_k \le \sup_k p_k = H < \infty$.

Theorem 2.5. *Let f be an unbounded modulus. The classes of sequences $w_\rho^{f,\alpha}(p)$ and $S_\rho^{f,\alpha}$ are linear spaces.*

Theorem 2.6. *The space $w_\rho^{f,\alpha}(p)$ is paranormed by*

$$g(x)=\sup_n \left\{ \frac{1}{f(\rho_n)^\alpha} \sum_{k=1}^n [f(|x_k|)]^{p_k} \right\}^{\frac{1}{M}}$$

where $0<\alpha\le 1$ and $M=\max(1,H)$.

Theorem 2.7. *Let f be an unbounded modulus, α be a real number such that $0<\alpha\le 1$ and $p_k=1$ for all $k\in\mathbb{N}$. If $\lim_{u\to\infty}\dfrac{f(u)^\alpha}{u^\alpha}=s>0\,(s\in\mathbb{R})$, then $w^\alpha[\rho,f]\subset S_\rho^{f,\alpha}$.*

Theorem 2.8. *Let α_1,α_2 be two real numbers such that $0<\alpha_1\le\alpha_2\le 1$ and f be an unbounded modulus function, then we have $w_\rho^{f,\alpha_1}(p)\subset S_\rho^{f,\alpha_2}$.*

Theorem 2.9. *Let α be a fixed real number such that $0<\alpha\le 1$. If $\liminf_n \dfrac{\rho_n}{n} > 1$ and $\lim_{u\to\infty}\dfrac{f(u)^\alpha}{u^\alpha}=s>0\,(s\in\mathbb{R})$, then $S^{f,\alpha}\subset S_\rho^{f,\alpha}$.*

Proof. Suppose first that $\liminf_n \dfrac{\rho_n}{n} > 1$; then there exists a $\lambda>0$ such that $\dfrac{\rho_n}{n}\ge 1+\lambda$ for sufficiently large n, which implies that

$$\frac{\rho_n}{n}\ge 1+\lambda \Longrightarrow \left(\frac{\rho_n}{n}\right)^\alpha \ge (1+\lambda)^\alpha$$

If $S^{f,\alpha}-\lim x_k=\ell$, then for every $\varepsilon>0$ and for sufficiently large n, we have

$$\begin{aligned}
&\frac{1}{f(n)^\alpha} f(|\{k\le n : |x_k-\ell|\ge\varepsilon\}|) \\
=\;& \frac{f(\rho_n)^\alpha}{f(n)^\alpha}\frac{1}{f(\rho_n)^\alpha} f(|\{k\le n : |x_k-\ell|\ge\varepsilon\}|) \\
=\;& \frac{f(\rho_n)^\alpha}{\rho_n^\alpha}\frac{n^\alpha}{f(n)^\alpha}\frac{\rho_n^\alpha}{n^\alpha}\frac{f(|\{k\le n : |x_k-\ell|\ge\varepsilon\}|)}{f(\rho_n)^\alpha} \\
\ge\;& \frac{f(\rho_n)^\alpha}{\rho_n^\alpha}\frac{n^\alpha}{f(n)^\alpha}(1+\lambda)^\alpha\frac{f(|\{k\le n : |x_k-\ell|\ge\varepsilon\}|)}{f(\rho_n)^\alpha}.
\end{aligned}$$

This proves the sufficiency. □

Theorem 2.10. *Let f be an unbounded modulus and $0 < \alpha \leq 1$. If $(x_k) \in S^f \cap S_\rho^{f,\alpha}$, then $S^f - \lim x_k = S_\rho^{f,\alpha} - \lim x_k$ such that $\lim_{n\to\infty} \dfrac{f(\rho_n)^\alpha}{f(n)} > 0$ and $|f(x) - f(y)| = f(|x-y|)$, for $x \geq 0$, $y \geq 0$.*

Proof. Suppose $S^f - \lim x_k = \ell_1$, $S_\rho^{f,\alpha} - \lim x_k = \ell_2$ and $\ell_1 \neq \ell_2$. Let $0 < \varepsilon < \frac{|\ell_1 - \ell_2|}{2}$. Then for $\varepsilon > 0$ we have

$$\lim_{n\to\infty} \frac{f(|\{k \leq n : |x_k - \ell_1| \geq \varepsilon\}|)}{f(n)} = 0,$$

and

$$\lim_{n\to\infty} \frac{f(|\{k \leq n : |x_k - \ell_2| \geq \varepsilon\}|)}{f(\rho_n)^\alpha} = 0.$$

On the other hand we can write

$$\begin{aligned} \frac{f(|\{k \leq n : |\ell_1 - \ell_2| \geq 2\varepsilon\}|)}{f(n)} &\leq \frac{f(|\{k \leq n : |x_k - \ell_1| \geq \varepsilon\}|)}{f(n)} \\ &+ \frac{f(|\{k \leq n : |x_k - \ell_2| \geq \varepsilon\}|)}{f(n)}. \end{aligned}$$

Taking limit as $n \to \infty$, we get

$$1 \leq 0 + \lim_{n\to\infty} \frac{f(|\{k \leq n : |x_k - \ell_2| \geq \varepsilon\}|)}{f(n)} \leq 1,$$

and so

$$\lim_{n\to\infty} \frac{f(|\{k \leq n : |x_k - \ell_2| \geq \varepsilon\}|)}{f(n)} = 1.$$

We have

$$\begin{aligned} &\frac{1}{f(n)} f(|\{k \leq n : |x_k - \ell_2| \geq \varepsilon\}|) \\ &= \frac{1}{f(n)} f(\rho_n)^\alpha \frac{1}{f(\rho_n)^\alpha} f(|\{k \leq n : |x_k - \ell_2| \geq \varepsilon\}|) \end{aligned}$$

so

$$\frac{1}{f(n)} f(|\{k \leq n : |x_k - \ell_2| \geq \varepsilon\}|) \to 0,$$

but this is a contradiction to

$$\lim_{n\to\infty}\frac{f\left(\left|\left\{k\leq n:|x_k-\ell_2|\geq\varepsilon\right\}\right|\right)}{f\left(n\right)}=1.$$

As a result, $\ell_1=\ell_2$. □

Now as a result of Theorem 2.10 we have the following Corollary 2.11.

Corollary 2.11. *Let* $\rho=(\rho_n)$ *and* $\rho'=(\rho'_n)$ *be two sequences as defined above and* $0<\alpha\leq 1$. *If* $(x_k)\in S^f\cap\left(S_\rho^{f,\alpha}\cap S_{\rho'}^{f,\alpha}\right)$, *then* $S_\rho^{f,\alpha}-\lim x_k=S_{\rho'}^{f,\alpha}-\lim x_k$.

Theorem 2.12. *Let* f *be an unbounded modulus. If* $\lim p_k>0$, *then* $w_\rho^{f,\alpha}(p)-\lim x_k=\ell$ *uniquely.*

Proof. Let $\lim p_k=s>0$. Assume that $w_\rho^{f,\alpha}(p)-\lim x_k=\ell_1$ and $w_\rho^{f,\alpha}(p)-\lim x_k=\ell_2$. Then

$$\lim_n\frac{1}{f\left(\rho_n\right)^\alpha}\sum_{k=1}^n\left[f\left(|x_k-\ell_1|\right)\right]^{p_k}=0,$$

and

$$\lim_n\frac{1}{f\left(\rho_n\right)^\alpha}\sum_{k=1}^n\left[f\left(|x_k-\ell_2|\right)\right]^{p_k}=0.$$

By definition of f, we have

$$\begin{aligned}&\frac{1}{f\left(\rho_n\right)^\alpha}\sum_{k=1}^n\left[f\left(|\ell_1-\ell_2|\right)\right]^{p_k}\\ \leq\;&\frac{D}{f\left(\rho_n\right)^\alpha}\left(\sum_{k=1}^n\left[f\left(|x_k-\ell_1|\right)\right]^{p_k}+\sum_{k=1}^n\left[f\left(|x_k-\ell_2|\right)\right]^{p_k}\right)\\ =\;&\frac{D}{f\left(\rho_n\right)^\alpha}\sum_{k=1}^n\left[f\left(|x_k-\ell_1|\right)\right]^{p_k}+\frac{D}{f\left(\rho_n\right)^\alpha}\sum_{k=1}^n\left[f\left(|x_k-\ell_2|\right)\right]^{p_k}\end{aligned}$$

where $\sup_k p_k=H$ and $D=\max\left(1,2^{H-1}\right)$. Hence

$$\lim_n\frac{1}{f\left(\rho_n\right)^\alpha}\sum_{k=1}^n\left[f\left(|\ell_1-\ell_2|\right)\right]^{p_k}=0.$$

Since $\lim_{k\to\infty}p_k=s$ we have $\ell_1-\ell_2=0$. Thus the limit is unique. □

Theorem 2.13. *Let $\rho = (\rho_n)$ and $\rho' = (\rho'_n)$ be two sequences as defined above such that $\rho_n \leq \rho'_n$ for all $n \in \mathbb{N}$ and α_1, α_2 two real numbers such that $0 < \alpha_1 \leq \alpha_2 \leq 1$. If*

$$\lim_{n\to\infty} \inf \frac{f(\rho_n)^{\alpha_1}}{f(\rho'_n)^{\alpha_2}} > 0 \tag{1}$$

then $w^{f,\alpha_2}_{\rho'}(p) \subset w^{f,\alpha_1}_{\rho}(p)$.

Proof. Let $x \in w^{f,\alpha_2}_{\rho'}(p)$. We can write

$$\frac{1}{f(\rho'_n)^{\alpha_2}} \sum_{k=1}^{n} [f(|x_k - \ell|)]^{p_k} \geq \frac{f(\rho_n)^{\alpha_1}}{f(\rho'_n)^{\alpha_2}} \frac{1}{f(\rho_n)^{\alpha_1}} \sum_{k=1}^{n} [f(|x_k - \ell|)]^{p_k}.$$

Thus if $x \in w^{f,\alpha_2}_{\rho'}(p)$, then $x \in w^{f,\alpha_1}_{\rho}(p)$. □

From Theorem 2.13 we have the following results.

Corollary 2.14. *Let $\rho = (\rho_n)$ and $\rho' = (\rho'_n)$ be two sequences as defined above such that $\rho_n \leq \rho'_n$ for all $n \in \mathbb{N}$ and α_1, α_2 two real numbers such that $0 < \alpha_1 \leq \alpha_2 \leq 1$. If (1) holds then*

(i) $w^{f,\alpha}_{\rho'}(p) \subset w^{f,\alpha}_{\rho}(p)$, *if* $\alpha_1 = \alpha_2 = \alpha$,

(ii) $w^{f}_{\rho'}(p) \subset w^{f,\alpha_1}_{\rho}(p)$, *if* $\alpha_2 = 1$,

(iii) $w^{f}_{\rho'}(p) \subset w^{f}_{\rho}(p)$, *if* $\alpha_1 = \alpha_2 = 1$.

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